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PROJECT: Hyperbolic extra-dimensional theory and phenomenology in particle physics and cosmology

Background and Rationale

While the Standard Model (SM) is a highly successful theoretical framework that can explain and predict — in exquisite detail and with razor-sharp precision – the origin and behaviour of all observed elementary particles, it has several limitations. One glaring example thereof is its inability to account for one of the four fundamental forces: gravity. At the subatomic level, the relative strengths of electromagnetism, the strong nuclear force, and the weak nuclear force vastly exceed gravity e.g. the electromagnetic repulsion between two protons (intrinsically positively-charged) is stronger than their gravitational attraction by a factor of 10^{36}. At present, there is no universally-accepted *a priori* justification for this "hierarchy problem".

However, to explain this imbalance in the scaling of the fundamental forces, several proposals have been put forth. Certain "extra-dimensional" (EDal) models, for example, consider gravity to appear weaker because its full effect is diluted across dimensions inaccessible to us. These consider the SM to exist on a 4D surface (comprised of three spatial directions and time as the fourth dimension) while gravity propagates throughout the full higher-dimensional space-time. In my project, we consider compact extra dimensions. A standard consequence of this compactification is higher-dimensional symmetry breaking, a remnant of which emerges as a parity, and the establishment of an infinite tower of standing waves in the EDs known as a Kaluza-Klein (KK) tower of states. Each state, labelled by an integer *n*, has an invariant mass $M_n \sim n/R$ where *R* is the radius of the extra compactified dimension.

As such, ED models serve as a promising cohort of beyond-the-standard-model (BSM) frameworks able to confront the hierarchy problem, as well as the dark matter and baryon asymmetry observations [1]. As TeV-scale effective theories, these ED models are experimentally accessible and have been the subject of numerous searches at high-energy particle collider experiments like the Large Hadron Collider (LHC), leading to improved constraints on the KK mass [1,2]. Statistically-significant evidence thereof, however, remains elusive.

Today, we find ourselves firmly ensconced within the era of gravitational-wave astronomy (GWA), with 83 confirmed black hole merger events detected to date by the LIGO-Virgo-KAGRA (LVK) collaboration [3]. These ripples in space-time, emergent from any accelerating massive body, are weakly-interacting – they pass through matter and radiation virtually unimpeded and unchanged. As such, they can ferry information from the furthest corners of the universe, allowing us to "see" beyond the limited range of the electromagnetic spectrum. GWs are thus complementary laboratories to collider experiments, as they can probe energies far beyond those accessible by machine and extend into the new territory within the parameter space of BSM models. As such, a growing body of research [4] indicates that GWA could be a feasible avenue through which to search for signatures of extra dimensions.

This project may then be considered a search for EDs through GWA. Our overarching aim is to assess the feasibility of the detection of a GW signature corresponding to an ED model whose extra spatial dimensions possess a negative scalar curvature. Of particular interest is the newly-proposed framework [5-7] whose ED geometry is a 3D "nilmanifold" – colloquially, a twisted torus. While the parameter space of flat and positively-curved EDs has been probed and constrained [8], models involving higher-dimensional manifolds (where the term "manifold" loosely refers to a surface) with negative curvature have remained under-explored. Negatively-curved ED models are usually confined to string theory and pure mathematics, where negative compact spaces have yielded a wealth of interesting results. Phenomenologically, studies on compact negative spaces are promising for their capacity to include cosmological observations such as homogeneity and flatness [9,10]. Moreover, these models could be used to address the hierarchy problem between the Planck and the electroweak scale by virtue of their geometrical properties [11].

Motivated by these, as well as the successful observation of post-merger GWs from black hole collisions known as "quasinormal frequencies" (QNFs) – complex frequencies $\omega = \omega_{Re} - i\omega_{Im}$ characteristic of their black hole source -- this project sets out to determine whether we can constrain ED models using GW data.

Objectives and methodology

1) To determine an effective 4D description of the Schwarzschild-nilmanifold setup

We embedded a 4D black hole into a 7D space-time whose higher dimensions form a 3D nilmanifold. By assuming the black hole metric to be independent of the nilmanifold coordinates, and vice-versa, we could express the extra-dimensional setup as a product space-time upon which the higher-dimensional field propagates. Due to the compactness of the higher-dimensional space, we could perform a "KK decomposition", such that the 7D field evolution is reduced to an effective 4D "massive" wave-like equation. The mass-like μ term then encoded the higher-dimensional behaviour. Finally, on the basis of spherical symmetry and harmonic time dependence, the equation of motion for the QNF could be conveyed through the radial component.

2) To compute the QNFs for the effective 4D BH

We used black hole perturbation theory and the semi-analytical Dolan-Ottewill expansion method [12] to express the QNF as a series of inverse angular numbers *l*. We applied this method here to the "massive" case, obtaining a series expansion in terms of μ and *l*, and compared our results against massive QNF computations performed in the literature [12].

3) To determine an upper bound for the detectability of EDs using QNFs

As a subset of scattering problems, QNF behaviour is defined by a "bell-shaped" barrier potential *V*. For $\mu = 0$, $V \to \infty$ as $r \to \infty$, and the effective potential has a distinct peak. For $\mu \neq 0$, $V \to \mu^2$ as $r \to \infty$; the curve is smoothed for increasing μ^2 and the QNF behaviour is lost. Beyond some critical μ, we therefore consider QNFs to be an inappropriate probe for extra dimensions.

4) To compare against LVK's most stringent bounds to constrain the detectability bound further

In their tests of general relativity (GR), the LVK collaboration searches for model-independent deviations from GR at the linear level in QNF spectra: $\omega_{Re} = \omega_{Re}^{GR}(1 + \Delta\omega_{Re})$. In their most recent catalogue [13], they reported a hierarchal combination of their strongest bounds to date, namely $\Delta\omega_{Re} = 0.02 \pm 0.07$ [14]. We considered $\omega_{Re}^{\mu=0} \approx \omega_{Re}^{GR}$, to use their result to solve for an upper bound on μ. This gave us a precise value for the critical μ at which QNF behaviour becomes suppressed and can no longer be relied upon as a probe of EDs.

Timeline

I first registered for my PhD at the University of Johannesburg (UJ) in July 2021; I then presented my PhD proposal at the UJ's Physics Department in January 2022, where it received approval at the month's end. My PhD is scheduled to be completed by the end of July 2024. This project in particular has been completed; the results have been published in Ref. [15]. Another project of mine is being prepared for submission and two more are in progress.

Results or preliminary data

We computed the QNFs using the Dolan-Ottewill method. The result is a series expansion where each term is a function of μ and *l* i.e. $\omega = \omega(\mu, l)$. For sufficiently small values of μ, we found excellent agreement with other techniques considered in the literature [13]. As suggested by the behaviour of the effective QNM potential, the method failed when $\mu > \omega_{Re}$, which we consider a consequence of the suppression of the QNF behaviour in this regime. From the QNF potential, the detectability bound determined was $\mu < 0.6$. Then, by using searches for parametric deviations from GR, we further constrained this probe to $\mu < 0.3681$. In determining an upper bound beyond which QNF behavior is lost, we obtained a tangible limit on the applicability of QNFs in detection strategies of EDs. We relate this numerical value to astrophysical black holes in Ref. [15].

References

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